Possible Self-Organised Criticality of snow avalanches.
Posible criticalidad auto-organizada de las avalanchas de nieve.

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Abstract: Distributions of snow avalanche starting zone sizes are shown to be scale invariant, in a same way as a number of other geophysical phenomena. In order to better understand the origin of this behaviour, we designed a cellular automaton that represents the interface between the substrate and the snow slab. Basal propagation of basal cracks followed by crown crack failures are reproduced. The scale invariance is also reproduced, provided the introduction of some heterogeneity in failure thresholds. The possible self-organised criticality of the release phenomenon is also analysed.

Keywords: avalanche triggering, scale invariance, cellular automata, self-organised criticality.

Resumen: Demostramos que las distribuciones de tamaños de avalanchas de placa tienen un carácter invariante de escala, como ya se averiguó para otros fenómenos geofísicos. Para comprender el origen de esta característica, construimos un autómata celular representando el interfaz entre la placa de nieve y el substrato. El autómata reproduce la propagación de la fisura basal y la apertura subsiguiente de una fisura de corona. También reproduce la invarianza de escala, pero unicamente si los umbrales de rotura se escogen de manera aleatoria. También se analiza el posible carácter de criticalidad auto-organizada del fenómeno.

Palabras clave: iniciación de avalanchas, invariancia de escala, autómatas celulares, criticalidad auto-organizada.

1. Introduction

Most studies on snow avalanches usually deal with avalanche flows rather than with mechanisms responsible for avalanche release. The latter were essentially investigated in the past using mechanics of defect-free continuum media. More recently the possible role of defects was introduced through basic fracture mechanics approaches (e.g. 1, 2). Though, the main problem that arises in the study of such natural phenomena is twofold: i) numerous parameters are involved, each of them being hardly accessible with a reasonable accuracy, and ii) the initial state of the system is not exactly known: snow packs never stop evolving owing to meteorological conditions, snow drift, etc. It therefore appears that a deterministic approach is probably not the best way to investigate such a complex problem.

The aim of this work is to study slab avalanche triggering on a different basis, using a probabilistic analysis. The scale invariant nature of snow avalanche release statistics was discovered recently for both avalanche acoustic emission and flow sizes (3), the latter point being further confirmed by another group (4). A similar scale invariance was also evidenced for avalanche starting zone sizes themselves, characterised by both slab widths (5) and crown crack depths (5, 6). This type of scaling was already known for other geological failures, as rockfalls, landslides or turbidites. Despite several attempts in simulating such phenomena, no satisfactory description was available so far, as the power law exponents given by these simulations did not usually reproduce field observations.

We report here some data on distributions of crown crack depths and slab widths. We describe a cellular automaton that reproduces the observed slab width scaling, and discuss the results in terms of the evolution of complex systems and of a possible self-organised criticality (SOC) of gravity-driven failures.

2. Field data analysis

In order to study slab avalanche release in a probabilistic way, we used a database of more than 5000 avalanches recorded on the La Plagne and Tignes ski resorts during 3 winters. These data contain lots of valuable informations such as avalanche triggering mode (artificial, accidental or natural), slab widths, crown crack heights, location, etc.

In the following, the avalanche starting zone size will be defined using either the slab width L or the crown crack depth H, as illustrated in fig. 1. All these data, plotted on the same diagrams, align on the same curve, whatever the triggering mode, the mountain range, or the gully they start from. Fig. 2 (top) shows that slab width distributions obey an approximate power law, without any roll-off at large scales. The plateau at small scales corresponds to the detection limit of small avalanches.

Fig. 1: Typical slab avalanche. The starting zone is characterised by both the crown crack depth H and the the crown crack length L.

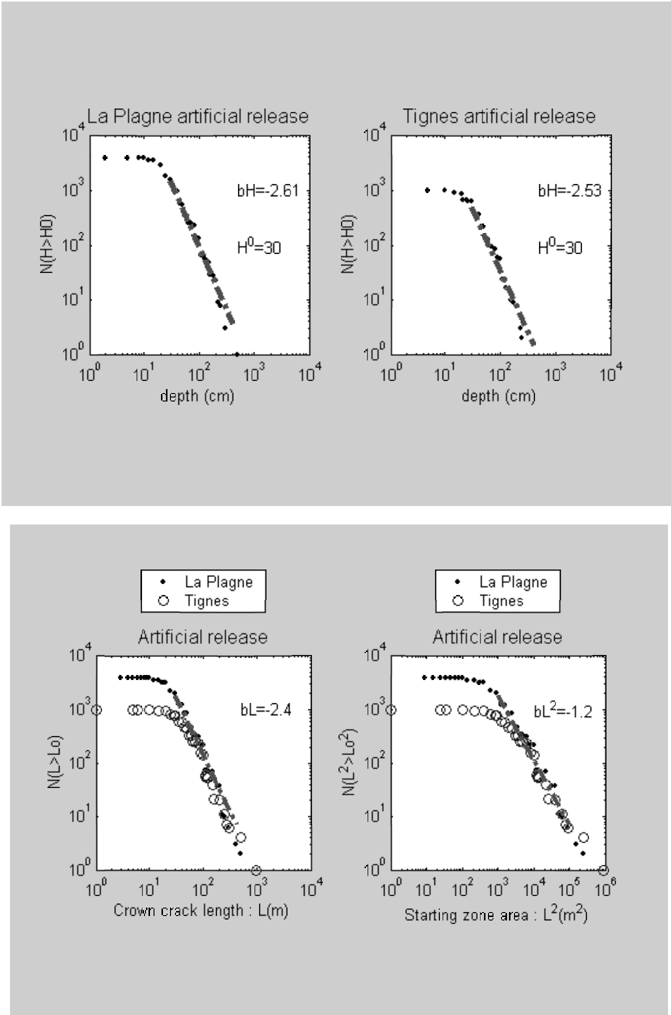

Figure 2: Cumulative distributions of avalanche sizes obtained from field data. Sizes can be represented by crown crack depths H (top figures) or lengths L (bottom figures). The corresponding exponents are between -2.5 and - 2.6 for crown crack depths, and around -2.4 for crown crack lengths. This latter result can be also expressed in terms of cumulative distributions of slab areas L^2, which gives an exponent -1.2, equivalent to -1.2 -1 = -2.2 in probability distribution of areas.

3. Simulations

In order to better understand the origin of such a possible scale invariance, we simulate avalanche release using cellular automata. This type of simulation is more or less similar to the so-called game of life, or to Bak's sand pile simulation (7) on which was based the concept of SOC, except that we introduce here two elementary mechanisms involved in slab release, instead of a single one: i) shear failure along a weak layer (basal crack), and ii) subsequent slab failure (crown crack opening). It is worth noting that, owing to its 2-dimensional character, a cellular automaton is not expected to give any information on crown crack depth statistics. In contrast, the use of such a 2-d automaton appears as a reasonable means of exploring the possible physical mechanisms leading to the observed distributions of starting zone areas. This is equivalent to studying slab widths, that scale as the square root of starting zone areas. In our simulation, a 2-d grid of cells represents the interface between the substrate and the snow slab, loaded in shear by the slab weight. Each cell can be found in one among 6 different states ξ (between 0 and 5) that represent the balance between the applied shear load and the local shear resistance. A snowfall is simulated by a gradual increase of cell states ξ scattered at random on the grid. Such an increase of ξ values may also represent a gradual weakening of the weak layer due to thermal transformation of snow crystals.

As slab avalanche release involves the expansion of a basal crack along a weak layer followed by the opening of a crown crack, two different rupture thresholds are introduced:

i) Shear failure: a cell fails in shear when its shear state ξ exceeds a threshold value τ_o. The mechanics of basal crack growth are taken into account through a local load transfer between neighbour cells: the load experienced by the failed cell before failure is equally redistributed to its neighbours. If such a redistribution brings the state of one of the neighbours up to or above the shear threshold, this neighbour fails in turn, and so on.

ii) Slab failure: if the shear stress experienced by the weak layer was uniform, there should not be any reason for crown crack opening. The stress in the slab itself results indeed from shear stress gradients along the weak layer. The second failure rule is therefore that a bond between two neighbour cells breaks if the difference $\delta\xi$ in the corresponding ξ values exceeds a slab failure threshold σ_o. The corresponding load is redistributed on the neighbour cells, except for the upper one.

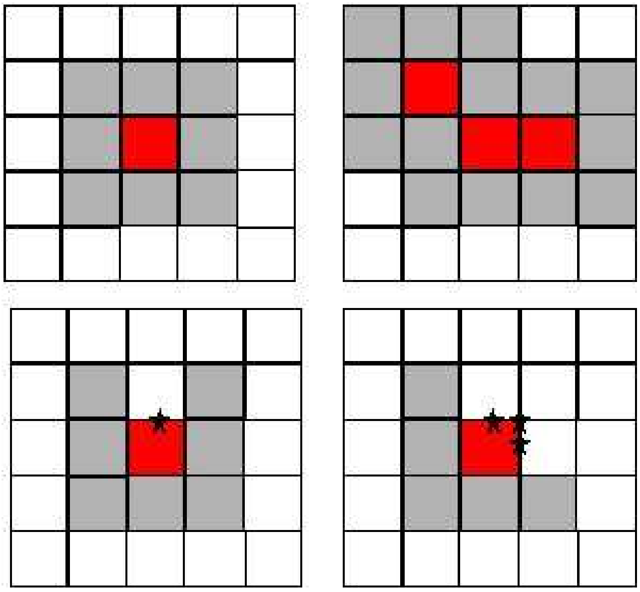

Fig. 3: Load redistribution rules. (a): shear failure: the load experienced by a cell (dark grey) that fails in shear is redistributed to its first neighbours, that may fail in turn. (b): slab failure: when a bond between two cells fails (stars), the load is redistributed to the first surviving neighbours, except for the one just above the failed bond.

4. Results:

The automaton can be run either from a non-loaded initial state followed by a random snowfall, or from a random initial load distribution followed by a gradual but uniform loading, which does not significantly change the results. During loading, many cell failure events result in collective failure cascades of various sizes, that represent a gradual damage of both the weak layer and the slab. A group of adjacent boxes in the ground state ("0" state) represents a basal crack. Each run eventually ends by a major failure event, within a single time step. The final configuration consists of a large cluster of damaged neighbour cells, that contains in its upper zone a smaller cluster of neighbour cells failed in shear, i.e. released from the substrate, but whose mutual bonds are still unbroken. This cluster represents the avalanche starting zone size, which will be used in the following statistics.

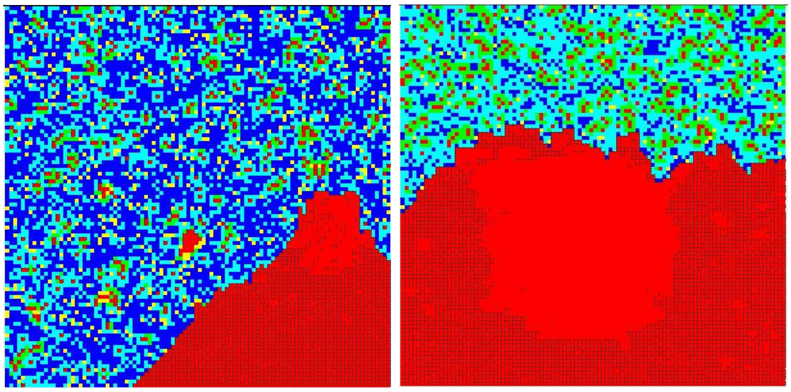

Figure 4: Natural avalanches simulated by a gradual cellular automaton. Box states vary from dark blue (undamaged) to bright red (totally cracked). The conditions are similar in both figures, except for the tensile rupture threshold which is small in the top figure and large in the bottom one. The starting zone corresponds to the uniformly red area.

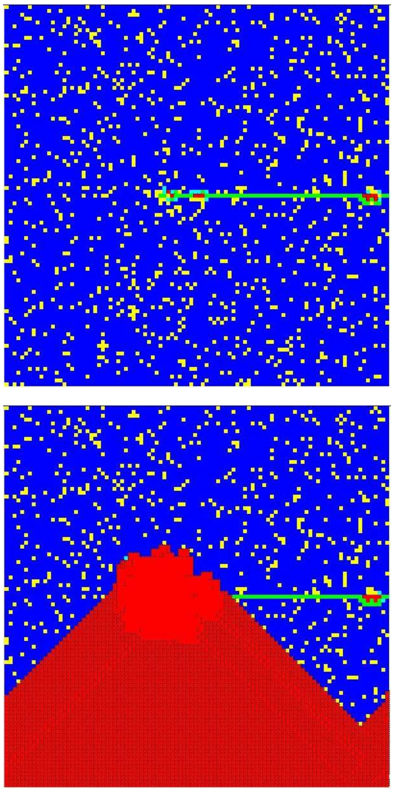

Fig. 5: Accidental avalanche: the skier comes from the right, and gradually damages the substrate. An avalanche is suddenly triggered from a starting zone that extends below and above the skiers position.

The obtained avalanches qualitatively reproduce field observations (fig. 4). We also used the automaton to model artificial triggerings: in this case, as a skier travels on the slab, cells are gradually "broken" (i.e. brought to the 0 state) along the skier's path (fig. 5). An avalanche is usually triggered after the skier has travelled some distance, as shown in fig. 5b. It is worth noticing that, for constant loading conditions and average slab strength, both avalanche triggering and starting zone location may considerably vary from one run to the next one, depending on the random choice of weak cells, and of spatial distribution and position of the skier's path. A skier may cross the whole slab without any triggering, whereas an avalanche may be triggered under identical average conditions, but a different random arrangement of cells. General trends can nevertheless be drawn from these simulations: the starting zone sizes, and therefore the avalanche sizes, increase with the slab tensile rupture strength, as expected intuitively: a larger damaged shear zone is necessary to lead to slab tensile failure as the slab failure stress threshold increases.

The automaton is run thousands of times, and the various starting zone sizes are recorded, as the number of cells belonging to the second cluster type, i.e. in terms of starting zone surfaces. However, if the slab failure threshold is not randomly distributed, but has a fixed value, the obtained size distributions are usually not scale invariant, but exhibit a characteristic scale. By contrast, if such a threshold is taken at random from a uniform distribution, starting zone size distributions are observed to be scale invariant (fig. 6), except for possible perturbations that may appear at large size values, related in most cases to box finite size effects. The exponent is found to depend on a single parameter $\alpha_{max} = (\sigma_o / \tau_o)_{max}$. The exponent obtained from field data is reproduced for $\alpha_{max} = 0.5$. This result suggests that snow may be more brittle in tension than in shear, which may appear all the more surprising at first glance as the shear resistance is that of the weak layer, whereas the "tensile" one refers to the usually stronger slab. However, this point is consistent with snow toughness measurements (8) that suggest that mode I (tensile) toughness may be significantly smaller than mode II (shear) toughness, possibly due to an easier healing of damaged bonds in shear than in tension. It also agrees with the particular loading state experienced by a snow slope, as recalled before: significant stresses within the slab only result from shear stress gradients along the slope. The former are therefore probably much weaker than the latter.

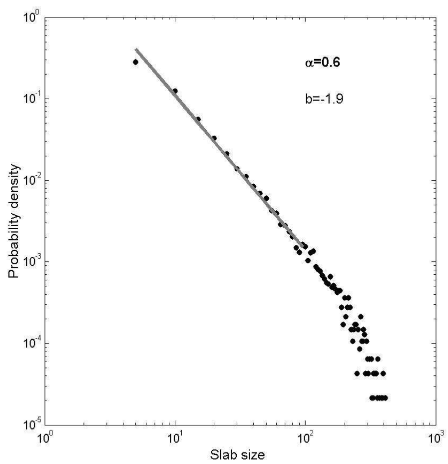

Fig. 6: Example of a slab area distribution obtained from the automaton for $\alpha_{max} = 0.6$. The exponent -b, with b = 1.9 obtained here in terms of area probability density would correspond in a cumulative distribution of slab widths to an exponent -b', with b' = 2(b-1) = 1.8, in good agreement with field data for rockfalls. The exponent characteristic of snow slab avalanche starting areas (b = 2.2) should be reproduced for $\alpha_{max} = 0.5$.

These results are quite different from those obtained from previous cellular automata simulation, as Bak's sand pile for instance. The main difference between them is that we introduce a slab threshold, i.e. that we take into account slab cohesion. We are thus able to deal with cohesive materials, in contrast with the assumption of a non cohesive dry sand in Bak's automaton. This is probably the reason for our larger exponents (in absolute values).

Another interesting point is that the exponents obtained from the automaton can be varied though slight tuning of the α_{max} parameter. The exponent values obtained from field data (expressed in probability distribution of areas), that are of about 1.75 for rockfalls (9), 2.8 for landslides (10) and 2.2 for snow avalanches, (fig. 2b), can be reproduced by tuning α_{max} in a narrow range between 0.5 and 1.

Our model therefore suggests that (i) cohesion is essential in understanding avalanche triggering mechanisms, (ii) the order of magnitudes of shear resistance (involved in load increments) and of tensile resistance should be similar and scattered, and (iii) this model may apply to a wider range of geophysical failures, provided the relative magnitudes and scattering of tensile and shear resistances are carefully tuned.

5. Discussion

Such scale invariant behaviours are often referred to as a possible self-organised criticality of the systems, that may emerge from long range non linear interactions between objects. The concept of self-organised criticality was introduced by Bak and co-workers (7) in order to describe the various relaxation events (called avalanches) occurring during loading, that keep the system around a steady state. The scale invariant statistics obtained in this case are thought to result from the system memory, each avalanche flow leaving a heterogeneous pattern after its occurrence. In our case, the equivalent of these relaxation events are the various damage cascades observed within a single run, corresponding to damage processes that are not directly observable in real life. By contrast, in order to describe the size distributions of snow avalanches, we are not interested here in the statistics of these damage cascades, but in those of the major failure events recorded at the end of each run.

More precisely, an essential difference between Bak' s sand pile system and ours is that the former is strain-rate driven, whereas the latter is stress-rate driven. In other words, the various relaxation events recorded in Bak' s simulation compensate the energy input, which allows the system to keep an average stationary energy balance. An obvious example is that of sliding of tectonic plates against one another, at roughly constant imposed shear strain rate, that results in a series of earthquakes of various magnitudes: each of them suddenly reduces the amount of elastic energy continuously stored by the displacement of tectonic plates, that is gradually restored by the plate motion. Our case is totally different, in that the load stemming from the snow weight is not relaxed by the successive cascades observed during a single run, but only redistributed to (and hence concentrated on) the neighbouring cells. The average load experienced by surviving cells increases thus gradually with snow storms, i.e. with time. Instead of evolving around a self-organised steady state, the system is necessarily brought to a final major failure event that totally relaxes the stored energy. In our case, the system is reinitialised to zero between runs, in order to describe "real life" independent avalanches that occur in different paths and winter seasons.

Since the various final avalanches are totally independent from one another, our scaling does not stem from self-organised criticality in its original formulation, and our model may be considered as an alternative to SOC. The scaling of such stress-driven systems may have an origin in space and time heterogeneities rather than in the system memory. This is suggested by the necessity of introducing randomness in failure thresholds in order to obtain scale-invariant distributions. In real life, this randomness probably arises from both the topography of the various avalanche paths, and the defect structure of the different snowpacks deposited during successive snowstorms. The field observation that all data retrieved from different seasons and avalanche paths align on the same power law may be considered as some kind of ergodicity.

6. Conclusions

Field data confirm the scale invariant (i.e. power law) distributions of slab avalanche sizes, described in terms of slab widths and surfaces. This is also the case for many other geophysical phenomena such as earthquakes, landslides, rockfalls, *etc.* This behaviour may be characteristic of complex systems interacting in a non linear manner. Simulations, and more particularly cellular automata, give interesting qualitative visualisations of avalanche triggering events, from which some qualitative trends can be drawn about the influence of external forcing (e.g. skier) on the triggering event, or the influence of the slab tensile strength on the avalanche size. They also reproduce the scale invariance observed from field data, provided some randomness is introduced in failure thresholds. The model also reproduces the scaling characteristics of other gravitational failures as landslides, turbidites or rockfalls, by tuning within a very narrow range a physically based parameter that may be understood as the cohesive anisotropy of the material.

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